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Determination of QSS by Single – Normal and Double Tightened plan using Fuzzy Binomial Distribution

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Abstract - This paper introduces a new sampling system named Fuzzy Quick Switching Single Double Sampling System which has two reference plans. FQSSDSS is a sampling system with reference to single sampling plan as normal and double sampling plan as tightened plan by incorporating a switching rule. This special type of Quick Switching System can be applied to protect from poor quality which give high level of protection and to reduce the cost of inspection. The concept of fuzzy logic is applied in this special sampling system solve the situations of fraction defective or Uncertainty or vagueness present in the parameter. This paper studies the determination of Quick Switching Single Double Sampling System using fuzzy Binomial distribution and the Acceptance number tightening method is followed. The table values and OC Curves of Normal, Tightened and FQSSDSS are also provided.

Keywords: Quick Switching System; Single Sampling Plan; Double Sampling Plan; Acceptance Number Tightening; Fuzzy Logic; Binomial Distribution; Probability of Acceptance; OC Curve.

1. INTRODUCTION AND REVIEW OF LITERATURE

This article shows how to evaluate and select quick switching system (QSSs) with single normal plan and double tightened plan. It introduces a new method of describing the high level production provided by sampling plans during periods of changing quality called transitive operating characteristic (OC) curves. This plan also adopts the situations of change of quality of the product.

The Quick Switching System explored in this article consists of two different reference sampling plans along with a set of rules for switching between them. This system starts with the plan called Normal plan which is used during the periods of good quality. It is one of the easiest methods to implement with smaller sample size which helps to reduce the sampling cost and time. When the problems are found in the product during the period of sampling the plan named Tightened plan can be implemented. This plan is specially designed to give a high level of protection. This plan reacts very quickly when the quality of the products got changed. QSSs were originally proposed by Dodge (1967) and investigated by Romboski (1969) and Govindaraju (1991). Taylor (1992) contains tables of QSSs and a program to select and evaluate QSSs with some modifications.

Soundarajan and Arumainayagam (1995) have analyzed QSS-r, r=1,2,3 with reference plans as single sampling plan, double sampling plan, chain sampling plan and repetitive group sampling plan . Arumainayagam and Uma.G (2009) studied QSS-r, r=1, 2, 3 with three stage multiple sampling plan as a reference plan. Arumainaygam and Vennila (2017) have applied two different types of reference plans in QSS-1 and find that the resulting system is more advantages than the system using same reference plan for normal and tightened inspection.

2. QUICK SWITCHING SINGLE DOUBLE SAMPLING SYSTEM

Quick switching system adopts normal – Single sampling plan when the quality is good and tightened – Double Sampling plan when the quality is bad. Dodge (1967) proposed a new sampling system consisting of pairs of normal and tightened plans with the specified switching rules constitute a sampling system. The application of system is as follows.

1. Adopt a pair of sampling plans, a normal plan (N) and tightened plan (T) the plan to T to be tighter ‘OC’ wiser than plan N
2. Use plan N for the first lot

3. For each lot inspected: if the lot is accepted, use plan N for the next lot; and if the lot is rejected, use plan T for the next lot.

Due to instantaneous switching between normal and tightened plan, the system is referred to as Quick Switching Single Double Sampling System. Using the OC function of QSSDSS is derived by Romboski (1969) as

$$P_a = \frac{P_T}{(1-P_N)+P_T} \quad (1)$$

4. ACCEPTANCE SAMPLING AND FUZZY SET THEORY:

In different acceptance sampling plans the fraction of defective items, is considered as a crisp value, but in practice the fraction of defective items value must be failed to known exactly. Many times these values are estimated or it is provided by experiment. When the vagueness present in the value of *p* by personal judgment, experiment or estimation the Fuzzy logic is the best one to solve this situation. As known, fuzzy set theory is powerful mathematical tool for modeling uncertain resulting. In this basis defining the imprecise proportion parameter is as a fuzzy number. With this definition, the number of nonconforming items in the sample has a Binomial distribution with fuzzy parameter. However if fuzzy number *p* is small we can use the fuzzy Poisson distribution to approximate values of the fuzzy Binomial.

5. DEFINITIONS:

Parameter 'p' (probability of a success) of the crisp Binomial distribution is known exactly, but sometimes we are failed to obtain the exact value of 'p', some uncertainty in the value 'p' and is to be estimated from a random sample or from expert opinion. The crisp Poisson distribution has one parameter, which we also assume is not known exactly.

Definition 5.1: The fuzzy subset $\widetilde{N}$ of real line *IR*, with the membership function $\mu_N : IR \rightarrow [0,1]$ is a fuzzy number if and only if (a) $\widetilde{N}$ is normal (b) $\widetilde{N}$ is fuzzy convex (c) μ_N is upper semi continuous (d) supp ($\widetilde{N}$) is bounded.

Definition 5.2: A triangular fuzzy number $\widetilde{N}$ is fuzzy number that membership function defined by three numbers $a_1 < a_2 < a_3$ where the base of the triangle is the interval $[a_1, a_3]$ and vertex is at $x = a_2$.

Definition 5.3: The α - cut of a fuzzy number $\widetilde{N}$ is a non-fuzzy set defined as

$$N[\alpha] = \{x \in IR; \mu_N(x) \geq \alpha\}.$$

Hence $N[\alpha] = [N_\alpha^L, N_\alpha^U]$ where

$$N_\alpha^L = inf\{x \in IR; \mu_N(x) \geq \alpha\}$$
$$N_\alpha^U = sup\{x \in IR; \mu_N(x) \geq \alpha\}$$

Definition 5.4: Due to the uncertainty in l_i's values we substitute $\tilde{l}_i$, a fuzzy number, for each l_i and assume that 0< $\tilde{l}_i$<1 all i . Then X together with the $\tilde{l}_i$value is a discrete fuzzy probability distribution. We write $\tilde{p}$ for fuzzy P and we have $\tilde{P}(\{x_i\}) = \tilde{l_\iota}$

Let A= $\{x_1, x_2, \ldots . x_l\}$ be subset of X. Then define:

$$\widetilde{P}\,(A)[\alpha] = \frac{\sum_{i=1}^{l} l_i}{s} \quad (2)$$

For 0<α< 1, where stands for the statement "$l_i \in \widetilde{k_\iota}$ [α], 1<i<n, $\sum_{i=1}^{l} l_i = 1$"
This is our fuzzy arithmetic.

Definition 5.5: Let *x* be a random variable having the Poisson mass function. If *P*(*x*) stands for the probability that X= x, then

$$\tilde{P}(d)[\alpha] = P(x \leq c)$$
$$= \sum_{d=0}^{c} \binom{n}{d} p^d (1-p)^{n-d} \quad (3)$$

Where p ϵ $\tilde{p}[1]$ and a q ϵ $\tilde{q}[1]$ with p+q = 1 and $\binom{n}{x} = \frac{m!}{r1\,(m-r)!}$

Then,

P^L [α] = min { $\binom{n}{d} p^d (1-p)^{n-d}$} and

P^U [α] = max { $\binom{n}{d} p^d (1-p)^{n-d}$}

The Fuzzy QSSDSS follows the two methods of tightening procedures with Normal – Single sampling plan and Tightened – Double sampling plan. They are:

- Acceptance number tightening
- Sample size tightening

For the above two systems following conditions are imposed.

$$(1) c_N > c_T \;\; \& \; (2)\; k > 1$$

When $c_N = c_T$ and k=1, the above two systems degenerate into single sampling plan.

Due to instantaneous switching between normal and tightened plan, the system is referred to as Quick Switching System. Using the OC function of QSS is derived by Romboski (1969) as

$$\boldsymbol{P_a} = \frac{\boldsymbol{P_T}}{(\boldsymbol{1}-\boldsymbol{P_N})+\boldsymbol{P_T}} \qquad \textbf{(4)}$$

P_N = Probability of Acceptance at Normal Level
P_T Probability of Acceptance at Tightened Level

6. ACCEPTANCE SAMPLING PLANS WITH FUZZY PARAMETER USING BINOMIAL DISTRIBUTION

In this section, the single sampling plan for classical attributes characteristics with fuzzy logic is introduced. Suppose to inspect a lot of size 'N', choose and inspect a random sample of size 'n', and count the number of defective items or damaged units (D). If the number of observed defective items (d) is less than or equal to the acceptance number 'c', the lot is accepted and otherwise it is rejected. If the size of the lot is very large, the random variable 'D' has a binomial distribution with parameters 'n' and 'p', where 'p' is the proportion of the defective items in the lot. So, the probability for the number of defective items to exactly equal 'd' is

$$P(D=d) = \binom{n}{\mathrm{d}} p^d (1-p)^{n-d} \qquad (5)$$

and hence the probability of acceptance of the lot is

$$P_a = P(D \le c) = \sum_{d=0}^{\mathrm{c}} \binom{n}{\mathrm{d}} p^d (1-p)^{n-d} \; (6)$$

Suppose to inspect a lot of size of 'N', where the proportion of defective items is not known precisely then suppose that this parameter is the fuzzy number $\tilde{p}$ as follows:

$$\tilde{p} = (a_1, a_2, a_3, a_4)\ ,$$

$$\tilde{p}[\alpha] = [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha] \qquad (7)$$

6.1 The Implementation of the Fuzzy Single Sampling Plan as Normal Plan:

A single sampling plan with a fuzzy parameter is defined by the sample size 'n', and acceptance number 'c', and if the number of observed defective items (d) is less than or equal to 'c', the lot will be accepted. If 'N' is a large number, then the number of defective items in this sample has a fuzzy binomial probability distribution. So for $0 \le \alpha \le 1$, the fuzzy probability that there will be exactly 'd' defective items in a sample of size 'n', is

$$\tilde{P}(D=d)[\alpha] = \left\{ \binom{n}{d} p^d q^{n-d} \,\middle|\, S \right\}$$

$$= [P^L[\alpha], P^U[\alpha]] \qquad (8)$$

$$P^L[\alpha] = min\left\{ \binom{n}{d} p^d q^{n-d} \,\middle|\, S \right\}$$

and $P^U[\alpha] = max\left\{ \binom{n}{d} p^d q^{n-d} \,\middle|\, S \right\}$ (9)

where S stands for the statement "$p \in \tilde{p}[\alpha]$, $q \in \tilde{q}[\alpha]$, p + q = 1"

6.2 The Implementation of Double Sampling Plan as Tightened Plan:

The parameters of the Double Sampling Plan is (n_1,p) and (n_2p) in which p indicates the fraction of nonconforming items of the lot, however, if the size of the sample be large and p is small then the random variable D_1 and D_2 has a Poisson approximation distribution with parameter $\lambda_1 = n_1 p$ and $\lambda_2 = n_2 p$. The Probability of acceptance for the Double sampling plan is the Probability of acceptance on combined samples with p_α and then the probability of the lot's acceptance in first and second samples by p_a^I, p_a^{II} , respectively , then,

$$p_a = p_a^I + p_a^{II} \qquad (10)$$

Where p_a^I indicates the probability of the event D1 = c1

Thus,

$$p_a^I = \left\{ \sum_{d_1=0}^{c_1} \binom{n}{\mathrm{d}} p^d (1-p)^{n-d}] \right\}$$

$$p_a^{II} = \mathrm{p}(D_1 + D_2 < c_2,\ c_1 < D_1 < c_2) \qquad (11)$$

The Fuzzy Probability of Acceptance of the lot in the first and second samples, $\tilde{p}_a^I$, $\tilde{p}_a^{II}$, respectively and

$$\tilde{p}_a = \tilde{p}_a^I + \tilde{p}_a^{II} \qquad (12)$$

7. OC – BAND WITH FUZZY PARAMETER

By the Operating Characteristic curve, one could determine the probability of acceptance or rejection of a lot having some specific defective items. One can understand the performance of the acceptance sampling plans and systems. OC curve can be drawn by plotting the probability of acceptance a lot versus its production quality, which is expressed by the proportion of non-conforming items in the lot. While applying the Fuzzy logic on

Acceptance sampling plans and Systems one can obtain the OC band rather than the OC curve which contains upper and lower OC Curves to determine the uncertainty levels. This OC band gives the clear vision about selection of plans that are effective in reducing risk and indicates discriminating power of the system.

The fuzzy probability of acceptance a lot in terms of fuzzy fraction of defective items would be as a band with upper and lower bounds under normal and tightened plans with varied sample sizes,

$$P^L[\alpha] = min\left\{\binom{n}{d} p^d q^{n-d} \,\middle|\, S\right\}$$ and

$$P^U[\alpha] = max\left\{\binom{n}{d} p^d q^{n-d} \,\middle|\, S\right\} \quad (13)$$

The uncertainty degree of proportion can be identified by the bandwidth of the OC curve. The less uncertainty value results in less bandwidth, and if proportion parameter gets a crisp value, lower and upper bounds will become equal, which that OC curve is in classic state.

Table 1: Probability of Acceptance for Fuzzy QSSDSS
SSP – (80,5) DSP – (80, 3,1) FQSSDSS – 3 (80, 5,3)

P	P_N	P_T	P(a)p
[0,0.01]	[1,0.9607]	[1,0.9259]	[1,0.9593]
[0.01,0.02]	[0.9986,0.8552]	[0.9259,0.6688]	[0.9986,0.8220]
[0.02,0.03]	[0.9766,0.7538]	[0.7323,0.4032]	[0.9690,0.6210]
[0.03,0.04]	[0.9034,0.6452]	[0.4487,0.2159]	[0.8229,0.3782]
[0.04,0.05]	[0.7763,0.5223]	[0.2375,0.1072]	[0.5150,0.1833]
[0.05,0.06]	[0.6189,0.3968]	[0.1155,0.0510]	[0.2326,0.0779]
[0.06,0.07]	[0.4606,0.2836]	[0.0537,0.0237]	[0.0906,0.0320]
[0.07,0.08]	[0.3226,0.1917]	[0.0245,0.0109]	[0.0349,0.0133]
[0.08,0.09]	[0.2143,0.1234]	[0.0111,0.0049]	[0.0139,0.0056]
[0.09,0.1]	[0.1359,0.0760]	[0.0050,0.0022]	[0.0057,0.0024]
[0.1,0.11]	[0.0827,0.0451]	[0.0020,0.0010]	[0.0024,0.0010]

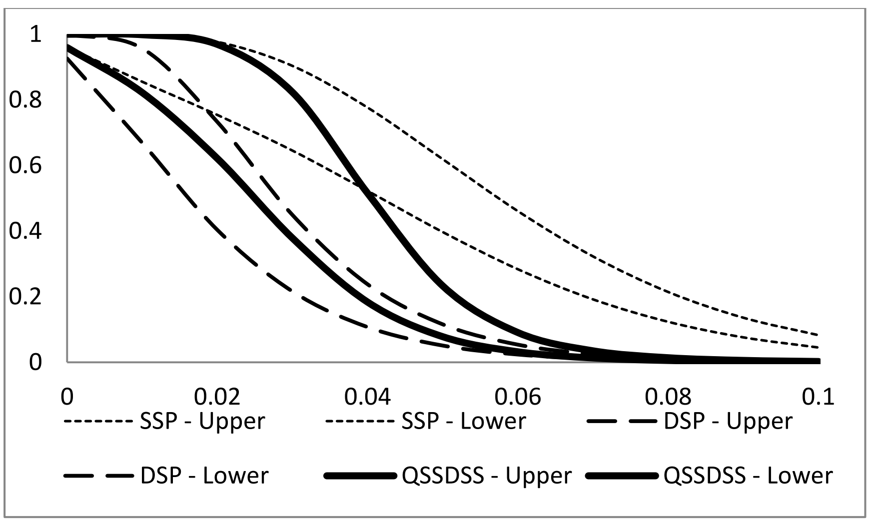

Figure 1: Probability of Acceptance for Fuzzy QSSDSS
SSP – (80,5) DSP – (80, 3,1) FQSSDSS – 3 (80, 5,3)

8. CONCLUSION

In this article the Construction and designing of FQSSDSS with reference to single normal plan and reference to double tightened plan using Fuzzy Binomial Distribution is provided the Probability of Acceptance table and OC band also studied. The uncertainty degree of a proportion parameter is one of the factors of the width of the OC band. The less uncertainty value results in less bandwidth, and greater uncertainty values results in wider bandwidth.

By the way of the FQSSDSS the high level prediction can be done on the uncertainty level. The FQSSDSS is highly useful method which stands forward with the better outcome with minimum sampling cost and time, also easiest to adopt while getting changes in the quality of the product. The Fuzzy QSSDSS is a versatile and valuable new tool for one's inspection program in shop floor situations with least samples, less cost and less time but more protection on quality.

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